

*The Origin of Geometry*¹

THE INTEREST THAT propels us in this work makes it necessary to engage first of all in reflections which surely never occurred to Galileo. We must focus our gaze not merely upon the ready-made, handed-down geometry and upon the manner of being which its meaning had in his thinking; it was no different in his thinking from what it was in that of all the late inheritors of the older geometric wisdom, whenever they were at work, either as pure geometers or as making practical applications of geometry. Rather, indeed above all, we must also inquire back into the original meaning of the handed-down geometry, which continued to be valid with this very same meaning—continued and at the same time was developed further, remaining simply “geometry” in all its new forms. Our considerations will necessarily lead to the deepest problems of meaning, problems of science and of the history of science in general, and indeed in the end to problems of a universal history in general; so that our problems and expositions concerning Galilean geometry take on an exemplary significance.

Let it be noted in advance that, in the midst of our historical meditations on modern philosophy, there appears here for the first time with Galileo, through the disclosure of the depth-problems of the meaning-origin of geometry and, founded on this, of the meaning-origin of his new physics, a clarifying light for our whole undertaking: namely, [the idea of] seeking to carry out, in the form of historical meditations, self-reflections about our own present philosophical situation in the hope that in this way we can finally take possession of the meaning, method, and beginning of philosophy, the *one* philosophy to which our life seeks to be and ought to be devoted. For, as will become evident here, at first in connection with one example, our investigations are historical in an unusual sense, namely, in virtue of a thematic direction which opens up depth-problems quite unknown to ordinary history, problems which, [however,] in their own way, are undoubtedly histori-

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cal problems. Where a consistent pursuit of these depth-problems leads can naturally not yet be seen at the beginning.

The question of the origin of geometry (under which title here, for the sake of brevity, we include all disciplines that deal with shapes existing mathematically in pure space-time) shall not be considered here as the philological-historical question, i.e., as the search for the first geometers who actually uttered pure geometrical propositions, proofs, theories, or for the particular propositions they discovered, or the like. Rather than this, our interest shall be the inquiry back into the most original sense in which geometry once arose, was present as the tradition of millennia, is still present for us, and is still being worked on in a lively forward development;* we inquire into that sense in which it appeared in history for the first time—in which it had to appear, even though we know nothing of the first creators and are not even asking after them. Starting from what we know, from our geometry, or rather from the older handed-down forms (such as Euclidean geometry), there is an inquiry back into the submerged original beginnings of geometry as they necessarily must have been in their “primally establishing” function. This regressive inquiry unavoidably remains within the sphere of generalities, but, as we shall soon see, these are generalities which can be richly explicated, with prescribed possibilities of arriving at particular questions and self-evident claims as answers. The geometry which is ready-made, so to speak, from which the regressive inquiry begins, is a tradition. Our human existence moves within innumerable traditions. The whole cultural world, in all its forms, exists through tradition. These forms have arisen as such not merely causally; we also know already that tradition is precisely tradition, having arisen within our human space through human activity, i.e., spiritually, even though we generally know nothing, or as good as nothing, of the particular provenance and of the spiritual source that brought it about. And yet there lies in this lack of knowledge, everywhere and essentially, an implicit knowledge, which can thus also be made explicit, a knowledge of unassailable self-evidence. It begins with superficial commonplaces, such as: that everything traditional has arisen out of human activity, that accordingly past men and human civilizations existed, and among them their first inventors, who shaped the new out of materials at hand, whether raw or already spiritually shaped. From the superficial, however, one is led into the depths. Tradition is open in this general way to continued inquiry; and, if one consistently maintains

* So also for Galileo and all the periods following the Renaissance, continually being worked on in a lively forward development, and yet at the same time a tradition.

the direction of inquiry, an infinity of questions opens up, questions which lead to definite answers in accord with their sense. Their form of generality—indeed, as one can see, of unconditioned general validity—naturally allows for application to individually determined particular cases, though it determines only that in the individual that can be grasped through subsumption.

Let us begin, then, in connection with geometry, with the most obvious commonplaces that we have already expressed above in order to indicate the sense of our regressive inquiry. We understand our geometry, available to us through tradition (we have learned it, and so have our teachers), to be a total acquisition of spiritual accomplishments which grows through the continued work of new spiritual acts into new acquisitions. We know of its handed-down, earlier forms, as those from which it has arisen; but with every form the reference to an earlier one is repeated. Clearly, then, geometry must have arisen out of a *first* acquisition, out of first creative activities. We understand its persisting manner of being: it is not only a mobile forward process from one set of acquisitions to another but a continuous synthesis in which all acquisitions maintain their validity, all make up a totality such that, at every present stage, the total acquisition is, so to speak, the total premise for the acquisitions of the new level. Geometry necessarily has this mobility and has a horizon of geometrical future in precisely this style; this is its meaning for every geometer who has the consciousness (the constant implicit knowledge) of existing within a forward development understood as the progress of knowledge being built into the horizon. The same thing is true of every science. Also, every science is related to an open chain of the generations of those who work for and with one another, researchers either known or unknown to one another who are the accomplishing subjectivity of the whole living science. Science, and in particular geometry, with this ontic meaning, must have had a historical beginning; this meaning itself must have an origin in an accomplishment: first as a project and then in successful execution.

Obviously it is the same here as with every other invention. Every spiritual accomplishment proceeding from its first project to its execution is present for the first time in the self-evidence of actual success. But when we note that mathematics has the manner of being of a lively forward movement from acquisitions as premises to new acquisitions, in whose ontic meaning that of the premises is included (the process continuing in this manner), then it is clear that the *total* meaning of geometry (as a developed science, as in the case of every science) could not have been present as a project and then as mobile fulfillment at the beginning. A more primitive formation of meaning necessarily went

before it as a preliminary stage, undoubtedly in such a way that it appeared for the first time in the self-evidence of successful realization. But this way of expressing it is actually overblown. Self-evidence means nothing more than grasping an entity with the consciousness of its original being-itself-there [*Selbst-da*]. Successful realization of a project is, for the acting subject, self-evidence; in this self-evidence, what has been realized is there, *originaliter*, as itself.

But now questions arise. This process of projecting and successfully realizing occurs, after all, purely within the *subject* of the inventor, and thus the meaning, as present *originaliter* with its whole content, lies exclusively, so to speak, within his mental space. But geometrical existence is not psychic existence; it does not exist as something personal within the personal sphere of consciousness: it is the existence of what is objectively there for "everyone" (for actual and possible geometers, or those who understand geometry). Indeed, it has, from its primal establishment, an existence which is peculiarly supertemporal and which—of this we are certain—is accessible to all men, first of all to the actual and possible mathematicians of all peoples, all ages; and this is true of all its particular forms. And all forms newly produced by someone on the basis of pre-given forms immediately take on the same objectivity. This is, we note, an "ideal" objectivity. It is proper to a whole class of spiritual products of the cultural world, to which not only all scientific constructions and the sciences themselves belong but also, for example, the constructions of fine literature.* Works of this class do not, like tools (hammers, pliers) or like architectural and other such products, have a repeatability in many like exemplars. The Pythagorean theorem, [indeed] all of geometry, exists only once, no matter how often or even in what language it may be expressed. It is identically the same in the "original language" of Euclid and in all "translations"; and within each language it is again the same, no matter how many times it has been sensibly uttered, from the original expression and writing-down to the innumerable oral utterances or written and other documentations. The sensible utterances have spatiotemporal individuation in the world like all corporeal occurrences, like everything embodied in bodies as such; but this is not true of the spiritual form itself,

* But the broadest concept of literature encompasses them all; that is, it belongs to their objective being that they be linguistically expressed and can be expressed again and again; or, more precisely, they have their objectivity, their existence-for-everyone, only as signification, as the meaning of speech. This is true in a peculiar fashion in the case of the objective sciences: for them the difference between the original language of the work and its translation into other languages does not remove its identical accessibility or change it into an inauthentic, indirect accessibility.

which is called an "ideal object" [*ideale Gegenständlichkeit*]. In a certain way ideal objects do exist objectively in the world, but it is only in virtue of these two-leveled repetitions and ultimately in virtue of sensibly embodying repetitions. For language itself, in all its particularizations (words, sentences, speeches), is, as can easily be seen from the grammatical point of view, thoroughly made up of ideal objects; for example, the word *Löwe* occurs only once in the German language; it is identical throughout its innumerable utterances by any given persons. But the idealities of geometrical words, sentences, theories—considered purely as linguistic structures—are not the idealities that make up what is expressed and brought to validity as truth in geometry; the latter are ideal geometrical objects, states of affairs, etc. Wherever something is asserted, one can distinguish what is thematic, that about which it is said (its meaning), from the assertion, which itself, during the asserting, is never and can never be thematic. And what is thematic here is precisely ideal objects, and quite different ones from those coming under the concept of language. Our problem now concerns precisely the ideal objects which are thematic in geometry: how does geometrical ideality (just like that of all sciences) proceed from its primary intrapersonal origin, where it is a structure within the conscious space of the first inventor's soul, to its ideal objectivity? In advance we see that it occurs by means of language, through which it receives, so to speak, its linguistic living body [*Sprachleib*]. But how does linguistic embodiment make out of the merely intrasubjective structure the *objective* structure which, e.g., as geometrical concept or state of affairs, is in fact present as understandable by all and is valid, already in its linguistic expression as geometrical speech, as geometrical proposition, for all the future in its geometrical sense?

Naturally, we shall not go into the general problem which also arises here of the origin of language in its ideal existence and its existence in the real world grounded in utterance and documentation; but we must say a few words here about the relation between language, as a function of man within human civilization, and the world as the horizon of human existence.

Living wakefully in the world we are constantly conscious of the world, whether we pay attention to it or not, conscious of it as the horizon of our life, as a horizon of "things" (real objects), of our actual and possible interests and activities. Always standing out against the world-horizon is the horizon of our fellow men, whether there are any of them present or not. Before even taking notice of it at all, we are conscious of the open horizon of our fellow men with its limited nucleus of our neighbors, those known to us. We are thereby coconscious of the

men on our external horizon in each case as "others"; in each case "I" am conscious of them as "my" others, as those with whom I can enter into actual and potential, immediate and mediate relations of empathy; [this involves] a reciprocal "getting along" with others; and on the basis of these relations I can deal with them, enter into particular modes of community with them, and then know, in a habitual way, of my being so related. Like me, every human being—and this is how he is understood by me and everyone else—has his fellow men and, always counting himself, civilization in general, in which he knows himself to be living.

It is precisely to this horizon of civilization that common language belongs. One is conscious of civilization from the start as an immediate and mediate linguistic community. Clearly it is only through language and its far-reaching documentations, as possible communications, that the horizon of civilization can be an open and endless one, as it always is for men. What is privileged in consciousness as the horizon of civilization and as the linguistic community is mature normal civilization (taking away the abnormal and the world of children). In this sense civilization is, for every man whose we-horizon it is, a community of those who can reciprocally express themselves, normally, in a fully understandable fashion; and within this community everyone can talk about what is within the surrounding world of his civilization as objectively existing. Everything has its name, or is namable in the broadest sense, i.e., linguistically expressible. The objective world is from the start the world for all, the world which "everyone" has as world-horizon. Its objective being presupposes men, understood as men with a common language. Language, for its part, as function and exercised capacity, is related correlatively to the world, the universe of objects which is linguistically expressible in its being and its being-such. Thus men as men, fellow men, world—the world of which men, of which we, always talk and can talk—and, on the other hand, language, are inseparably intertwined; and one is always certain of their inseparable relational unity, though usually only implicitly, in the manner of a horizon.

This being presupposed, the primally establishing geometer can obviously also express his internal structure. But the question arises again: How does the latter, in its "ideality," thereby become objective? To be sure, something psychic which can be understood by others [*nachverstehbar*] and is communicable, as something psychic belonging to this man, is *eo ipso* objective, just as he himself, as concrete man, is experienceable and namable by everyone as a real thing in the world of

things in general. People can agree about such things, can make common verifiable assertions on the basis of common experience, etc. But how does the intrapsychically constituted structure arrive at an intersubjective being of its own as an ideal object which, as "geometrical," is anything but a real psychic object, even though it has arisen psychically? Let us reflect. The original being-itself-there, in the immediacy [*Aktualität*] of its first production, i.e., in original "self-evidence," results in no persisting acquisition at all that could have objective existence. Vivid self-evidence passes—though in such a way that the activity immediately turns into the passivity of the flowingly fading consciousness of what-has-just-now-been. Finally this "retention" disappears, but the "disappeared" passing and being past has not become nothing for the subject in question: it can be reawakened. To the passivity of what is at first obscurely awakened and what perhaps emerges with greater and greater clarity there belongs the possible activity of a recollection in which the past experiencing [*Erleben*] is lived through in a quasi-new and quasi-active way. Now if the originally self-evident production, as the pure fulfillment of its intention, is what is renewed (recollected), there necessarily occurs, accompanying the active recollection of what is past, an activity of concurrent actual production, and there arises thereby, in original "coincidence," the self-evidence of identity: what has now been realized in original fashion is the same as what was previously self-evident. Also coestablished is the capacity for repetition at will with the self-evidence of the identity (coincidence of identity) of the structure throughout the chain of repetitions. Yet even with this, we have still not gone beyond the subject and his subjective, evident capacities; that is, we still have no "objectivity" given. It does arise, however—in a preliminary stage—in understandable fashion as soon as we take into consideration the function of empathy and fellow mankind as a community of empathy and of language. In the contact of reciprocal linguistic understanding, the original production and the product of one subject can be *actively* understood by the others. In this full understanding of what is produced by the other, as in the case of recollection, a present coaccomplishment on one's own part of the presentified activity necessarily takes place; but at the same time there is also the self-evident consciousness of the identity of the mental structure in the productions of both the receiver of the communication and the communicator; and this occurs reciprocally. The productions can reproduce their likenesses from person to person, and in the chain of the understanding of these repetitions what is self-evident turns up as the same in the consciousness of the other. In the unity of the commu-

nity of communication among several persons the repeatedly produced structure becomes an object of consciousness, not as a likeness, but as the one structure common to all.

Now we must note that the objectivity of the ideal structure has not yet been fully constituted through such actual transferring of what has been originally produced in one to others who originally reproduce it. What is lacking is the *persisting existence* of the "ideal objects" even during periods in which the inventor and his fellows are no longer wakefully so related or even are no longer alive. What is lacking is their continuing-to-be even when no one has [consciously] realized them in self-evidence.

The important function of written, documenting linguistic expression is that it makes communications possible without immediate or mediate personal address; it is, so to speak, communication become virtual. Through this, the communalization of man is lifted to a new level. Written signs are, when considered from a purely corporeal point of view, straightforwardly, sensibly experienceable; and it is always possible that they be intersubjectively experienceable in common. But as linguistic signs they awaken, as do linguistic sounds, their familiar significations. The awakening is something passive; the awakened signification is thus given passively, similarly to the way in which any other activity which has sunk into obscurity, once associatively awakened, emerges at first *passively* as a more or less clear memory. In the passivity in question here, as in the case of memory, what is passively awakened can be transformed back,* so to speak, into the corresponding activity: this is the capacity for reactivation that belongs originally to every human being as a speaking being. Accordingly, then, the writing-down effects a transformation of the original mode of being of the meaning-structure, [e.g.,] within the geometrical sphere of self-evidence, of the geometrical structure which is put into words. It becomes sedimented, so to speak. But the reader can make it self-evident again, can reactivate the self-evidence.†

There is a distinction, then, between passively understanding the expression and making it self-evident by reactivating its meaning. But there also exist possibilities of a kind of activity, a thinking in terms of

* This is a transformation of which one is conscious as being in itself patterned after [what is passively awakened].

† But this is by no means necessary or even factually normal. Even without this he can understand; he can concur "as a matter of course" in the validity of what is understood without any activity of his own. In this case he comports himself purely passively and receptively.

things that have been taken up merely receptively, passively, which deals with significations only passively understood and taken over, without any of the self-evidence of original activity. Passivity in general is the realm of things that are bound together and melt into one another associatively, where all meaning that arises is put together passively. What often happens here is that a meaning arises which is apparently possible as a unity—i.e., can apparently be made self-evidence through a possible reactivation—whereas the attempt at actual reactivation can reactivate only the individual members of the combination, while the intention to unify them into a whole, instead of being fulfilled, comes to nothing; that is, the ontic validity is destroyed through the original consciousness of nullity.

It is easy to see that even in [ordinary] human life, and first of all in every individual life from childhood up to maturity, the originally intuitive life which creates its originally self-evident structures through activities on the basis of sense-experience very quickly and in increasing measure falls victim to the *seduction of language*. Greater and greater segments of this life lapse into a kind of talking and reading that is dominated purely by association; and often enough, in respect to the validities arrived at in this way, it is disappointed by subsequent experience.

Now one will say that in the sphere that interests us here—that of science, of thinking directed toward the attainment of truths and the avoidance of falsehood—one is obviously greatly concerned from the start to put a stop to the free play of associative constructions. In view of the unavoidable sedimentation of mental products in the form of persisting linguistic acquisitions, which can be taken up again at first merely passively and be taken over by anyone else, such constructions remain a constant danger. This danger is avoided if one not merely convinces oneself *ex post facto* that the particular construction can be reactivated but assures oneself from the start, after the self-evident primal establishment, of its capacity to be reactivated and enduringly maintained. This occurs when one has a view to the univocity of linguistic expression and to securing, by means of the most painstaking formation of the relevant words, propositions, and complexes of propositions, the results which are to be univocally expressed. This must be done by the individual scientist, and not only by the inventor but by every scientist as a member of the scientific community after he has taken over from the others what is to be taken over. This belongs, then, to the particulars of the scientific tradition within the corresponding community of scientists as a community of knowledge living in the unity of a common responsibility. In accord with the essence of science,

then, its functionaries maintain the constant claim, the personal certainty, that everything they put into scientific assertions has been said "once and for all," that it "stands fast," forever identically repeatable with self-evidence and usable for further theoretical or practical ends—as indubitably reactivatable with the identity of its actual meaning.*

However, two more things are important here. First: we have not yet taken into account the fact that scientific thinking attains new results on the basis of those already attained, that the new ones serve as the foundation for still others, etc.—in the unity of a propagative process of transferred meaning.

In the finally immense proliferation of a science like geometry, what has become of the claim and the capacity for reactivation? When every researcher works on his part of the building, what of the vocational interruptions and time out for rest, which cannot be overlooked here? When he returns to the actual continuation of work, must he first run through the whole immense chain of groundings back to the original premises and actually reactivate the whole thing? If so, a science like our modern geometry would obviously not be possible at all. And yet it is of the essence of the results of each stage not only that their ideal ontic meaning in fact comes later [than that of earlier results] but that, since meaning is grounded upon meaning, the earlier meaning gives something of its validity to the later one, indeed becomes part of it to a certain extent. Thus no building block within the mental structure is self-sufficient; and none, then, can be immediately reactivated [by itself].

This is especially true of sciences which, like geometry, have their thematic sphere in ideal products, in idealities from which more and more idealities at higher levels are produced. It is quite different in the so-called descriptive sciences, where the theoretical interest, classifying and describing, remains within the sphere of sense-intuition, which for it represents self-evidence. Here, at least in general, every new proposition can by itself be "cashed in" for self-evidence.

How, by contrast, is a science like geometry possible? How, as a systematic, endlessly growing stratified structure of idealities, can it maintain its original meaningfulness through living reactivatability if its

* At first, of course, it is a matter of a firm direction of the will, which the scientist establishes in himself, aimed at the certain capacity for reactivation. If the goal of reactivatability can be only relatively fulfilled, then the claim which stems from the consciousness of being able to acquire something also has its relativity; and this relativity also makes itself noticeable and is driven out. Ultimately, objective, absolutely firm knowledge of truth is an infinite idea.

cognitive thinking is supposed to produce something new without being able to reactivate the previous levels of knowledge back to the first? Even if this could have succeeded at a more primitive stage of geometry, its energy would ultimately have been too much spent in the effort of procuring self-evidence and would not have been available for a higher productivity.

Here we must take into consideration the peculiar "logical" activity which is tied specifically to language, as well as to the ideal cognitive structures that arise specifically within it. To any sentence structures that emerge within a merely passive understanding there belongs essentially a peculiar sort of activity best described by the word "explication."² A passively emerging sentence (e.g., in memory), or one heard and passively understood, is at first merely received with a passive ego-participation, taken up as valid; and in this form it is already our meaning. From this we distinguish the peculiar and important activity of explicating our meaning. Whereas in its first form it was a straightforwardly valid meaning, taken up as unitary and undifferentiated—concretely speaking, a straightforwardly valid declarative sentence—now what in itself is vague and undifferentiated is actively explicated. Consider, for example, the way in which we understand, when superficially reading the newspaper, and simply receive the "news"; here there is a passive taking-over of ontic validity such that what is read straightway becomes our opinion.

But it is something special, as we have said, to have the intention to explicate, to engage in the activity which articulates what has been read (or an interesting sentence from it), extracting one by one, in separation from what has been vaguely, passively received as a unity, the elements of meaning, thus bringing the total validity to active performance in a new way on the basis of the individual validities. What was a passive meaning-pattern has now become one constructed through active production. This activity, then, is a peculiar sort of self-evidence; the structure arising out of it is in the mode of having been originally produced. And in connection with this self-evidence, too, there is communalization. The explicated judgment becomes an ideal object capable of being passed on. It is this object exclusively that is meant by logic when it speaks of sentences or judgments. And thus the *domain of logic* is universally designated; this is universally the sphere of being to which logic pertains insofar as it is the theory of the sentences [or propositions] in general.

Through this activity, now, further activities become possible—self-

² *Verdeutlichung*, i.e., making explicit.

evident constructions of new judgments on the basis of those already valid for us. This is the peculiar feature of logical thinking and of its purely logical self-evidences. All this remains intact even when judgments are transformed into assumptions, where, instead of ourselves asserting or judging, we think ourselves into the position of asserting or judging.

Here we shall concentrate on the sentences of language as they come to us passively and are merely received. In this connection it must also be noted that sentences give themselves in consciousness as reproductive transformations of an original meaning produced out of an actual, original activity; that is, in themselves they refer to such a genesis. In the sphere of logical self-evidence, deduction, or inference in forms of consequence, plays a constant and essential role. On the other hand, one must also take note of the constructive activities that operate with geometrical idealities which have been explicated but not brought to original self-evidence. (Original self-evidence must not be confused with the self-evidence of "axioms"; for axioms are in principle already the results of original meaning-construction and always have this behind them.)

Now what about the possibility of complete and genuine reactivation in full originality, through going back to the primal self-evidences, in the case of geometry and the so-called "deductive" sciences (so called, although they by no means merely deduce)? Here the fundamental law, with unconditionally general self-evidence, is: if the premises can actually be reactivated back to the most original self-evidence, then their self-evident consequences can be also. Accordingly it appears that, beginning with the primal self-evidences, the original genuineness must propagate itself through the chain of logical inference, no matter how long it is. However, if we consider the obvious finitude of the individual and even the social capacity to transform the logical chains of centuries, truly in the unity of one accomplishment, into originally genuine chains of self-evidence, we notice that the [above] law contains within itself an idealization: namely, the removal of limits from our capacity, in a certain sense its infinitization. The peculiar sort of self-evidence belonging to such idealizations will concern us later.

These are, then, the general essential insights which elucidate the whole methodical development of the "deductive" sciences and with it the manner of being which is essential to them.

These sciences are not handed down ready-made in the form of documented sentences; they involve a lively, productively advancing formation of meaning, which always has the documented, as a sediment of earlier production, at its disposal in that it deals with it logically. But

out of sentences with sedimented signification, logical "dealing" can produce only other sentences of the same character. That all new acquisitions express an actual geometrical truth is certain a priori under the presupposition that the foundations of the deductive structure have truly been produced and objectified in original self-evidence, i.e., have become universally accessible acquisitions. A continuity from one person to another, from one time to another, must have been capable of being carried out. It is clear that the method of producing original idealities out of what is prescientifically given in the cultural world must have been written down and fixed in firm sentences prior to the existence of geometry; furthermore, the capacity for translating these sentences from vague linguistic understanding into the clarity of the reactivation of their self-evident meaning must have been, in its own way, handed down and ever capable of being handed down.

Only as long as this condition was satisfied, or only when the possibility of its fulfillment was perfectly secured for all time, could geometry preserve its genuine, original meaning as a deductive science throughout the progression of logical constructions. In other words, only in this case could every geometer be capable of bringing to mediate self-evidence the meaning borne by every sentence, not merely as its sedimented (logical) sentence-meaning but as its actual meaning, its truth-meaning. And so for all of geometry.

The progress of deduction follows formal-logical self-evidence; but without the actually developed capacity for reactivating the original activities contained within its fundamental concepts, i.e., without the "what" and the "how" of its prescientific materials, geometry would be a tradition empty of meaning; and if we ourselves did not have this capacity, we could never even know whether geometry had or ever did have a genuine meaning, one that could really be "cashed in."

Unfortunately, however, this is our situation, and that of the whole modern age.

The "presupposition" mentioned above has in fact never been fulfilled. How the living tradition of the meaning-formation of elementary concepts is actually carried on can be seen in elementary geometrical instruction and its textbooks; what we actually learn there is how to deal with *ready-made* concepts and sentences in a rigorously methodical way. Rendering the concepts sensibly intuitable by means of drawn figures is substituted for the actual production of the primal idealities. And the rest is done by success—not the success of actual insight extending beyond the logical method's own self-evidence, but the practical successes of applied geometry, its immense, though not understood, practical usefulness. To this we must add something that will

become visible further on in the treatment of historical mathematics, namely, the dangers of a scientific life that is completely given over to logical activities. These dangers lie in certain progressive transformations of meaning* to which this sort of scientific treatment drives one.

By exhibiting the essential presuppositions upon which rests the historical possibility of a genuine tradition, true to its origins, of sciences like geometry, we can understand how such sciences can vitally develop throughout the centuries and still not be genuine. The inheritance of propositions and of the method of logically constructing new propositions and idealities can continue without interruption from one period to the next, while the capacity for reactivating the primal beginnings, i.e., the sources of meaning for everything that comes later, has not been handed down with it. What is lacking is thus precisely what had given and had to give meaning to all propositions and theories, a meaning arising from the primal sources which can be made self-evident again and again.

Of course, grammatically coherent propositions and concatenations of propositions, no matter how they have arisen and have achieved validity—even if it is through mere association—have in all circumstances their own logical meaning, i.e., their meaning that can be made self-evident through explication; this can then be identified again and again as the same proposition, which is either logically coherent or incoherent, where in the latter case it cannot be executed in the unity of an actual judgment. In propositions which belong together in one domain and in the deductive systems that can be made out of them we have a realm of ideal identities; and for these there exist easily understandable possibilities of lasting traditionalization. But propositions, like other cultural structures, appear on the scene in the form of tradition; they claim, so to speak, to be sedimentations of a truth-meaning that can be made originally self-evident; whereas it is by no means necessary that they [actually] have such a meaning, as in the case of associatively derived falsifications. Thus the whole pre-given deductive science, the total system of propositions in the unity of their validities, is first only a claim which can be justified as an expression of the alleged truth-meaning only through the actual capacity for reactivation.

Through this state of affairs we can understand the deeper reason for the demand, which has spread throughout the modern period and has finally been generally accepted, for a so-called “epistemological

* These work to the benefit of logical method, but they remove one further and further from the origins and make one insensitive to the problem of origin and thus to the actual ontic and truth-meaning of all these sciences.

grounding” of the sciences, though clarity has never been achieved about what the much-admired sciences are actually lacking.*

As for further details on the uprooting of an originally genuine tradition, i.e., one which involved original self-evidence at its actual first beginning, one can point to possible and easily understandable reasons. In the first oral cooperation of the beginning geometers, the need was understandably lacking for an exact fixing of descriptions of the prescientific primal material and of the ways in which, in relation to this material, geometrical idealities arose together with the first “axiomatic” propositions. Further, the logical superstructures did not yet rise so high that one could not return again and again to the original meaning. On the other hand, the possibility of the practical application of the derived laws, which was actually obvious in connection with the original developments, understandably led quickly, in the realm of praxis, to a habitually practiced method of using mathematics, if need be, to bring about useful things. This method could naturally be handed down even without the ability for original self-evidence. Thus mathematics, emptied of meaning, could generally propagate itself, constantly being added to logically, as could the methodics of technical application on the other side. The extraordinarily far-reaching practical usefulness became of itself a major motive for the advancement and appreciation of these sciences. Thus also it is understandable that the lost original truth-meaning made itself felt so little, indeed, that the need for the corresponding regressive inquiry had to be reawakened. More than this: the true sense of such an inquiry had to be discovered.

Our results based on principle are of a generality that extends over all the so-called deductive sciences and even indicates similar problems and investigations for all sciences. For all of them have the mobility of sedimented traditions that are worked upon, again and again, by an activity of producing new structures of meaning and handing them down. Existing in this way, they extend enduringly through time, since all new acquisitions are in turn sedimented and become working materials. Everywhere the problems, the clarifying investigations, the insights of principle are *historical*. We stand within the horizon of human civilization, the one in which we ourselves now live. We are constantly, vitally conscious of this horizon, and specifically as a temporal horizon implied in our given present horizon. To the one human civilization there corresponds essentially the one cultural world as the surrounding life-world with its [peculiar] manner of being; this world, for every

* What does Hume do but endeavor to inquire back into the primal impressions of developed ideas and, in general, scientific ideas?

historical period and civilization, has its particular features and is precisely the tradition. We stand, then, within the historical horizon in which everything is historical, even though we may know very little about it in a definite way. But it has its essential structure that can be revealed through methodical inquiry. This inquiry prescribes all the possible specialized questions, thus including, for the sciences, the inquiries back into origin which are peculiar to them in virtue of their historical manner of being. Here we are led back to the primal materials of the first formation of meaning, the primal premises, so to speak, which lie in the prescientific cultural world. Of course, this cultural world has in turn its own questions of origin, which at first remain unasked.

Naturally, problems of this particular sort immediately awaken the total problem of the universal historicity of the correlative manners of being of humanity and the cultural world and the a priori structure contained in this historicity. Still, questions like that of the clarification of the origin of geometry have a closed character, such that one need not inquire beyond those prescientific materials.

Further clarifications will be made in connection with two objections which are familiar to our own philosophical-historical situation.

In the first place, what sort of strange obstinacy is this, seeking to take the question of the origin of geometry back to some undiscoverable Thales of geometry, someone not even known to legend? Geometry is available to us in its propositions, its theories. Of course we must and we can answer for this logical edifice to the last detail in terms of self-evidence. Here, to be sure, we arrive at first axioms, and from them we proceed to the original self-evidence which the fundamental concepts make possible. What is this, if not the "theory of knowledge," in this case specifically the theory of geometrical knowledge? No one would think of tracing the epistemological problem back to such a supposed Thales. This is quite superfluous. The presently available concepts and propositions themselves contain their own meaning, first as nonself-evident opinion, but nevertheless as true propositions with a meant but still hidden truth which we can obviously bring to light by rendering the propositions themselves self-evident.

Our answer is as follows. Certainly the historical backward reference has not occurred to anyone; certainly theory of knowledge has never been seen as a peculiarly historical task. But this is precisely what we object to in the past. The ruling dogma of the separation in principle between epistemological elucidation and historical, even humanistic-psychological explanation, between epistemological and genetic origin, is fundamentally mistaken, unless one inadmissibly limits, in the usual way, the concepts of "history," "historical explanation," and

"genesis." Or rather, what is fundamentally mistaken is the limitation through which precisely the deepest and most genuine problems of history are concealed. If one thinks over our expositions (which are of course still rough and will later of necessity lead us into new depth-dimensions), what they make obvious is precisely that what we know—namely, that the presently vital cultural configuration "geometry" is a tradition and is still being handed down—is not knowledge concerning an external causality which effects the succession of historical configurations, as if it were knowledge based on induction, the presupposition of which would amount to an absurdity here; rather, to understand geometry or any given cultural fact is to be conscious of its historicity, albeit "implicitly." This, however, is not an empty claim; for quite generally it is true for every fact given under the heading of "culture," whether it is a matter of the lowliest culture of necessities or the highest culture (science, state, church, economic organization, etc.), that every straightforward understanding of it as an experiential fact involves the "coconsciousness" that it is something constructed through human activity. No matter how hidden, no matter how merely "implicitly" coimplied this meaning is, there belongs to it the self-evident possibility of explication, of "making it explicit" and clarifying it. Every explication and every transition from making explicit to making self-evident (even perhaps in cases where one stops much too soon) is nothing other than historical disclosure; in itself, essentially, it is something historical, and as such it bears, with essential necessity, the horizon of its history within itself. This is of course also to say that the whole of the cultural present, understood as a totality, "implies" the whole of the cultural past in an undetermined but structurally determined generality. To put it more precisely, it implies a continuity of pasts which imply one another, each in itself being a past cultural present. And this whole continuity is a *unity* of traditionalization up to the present, which is our present *as* [a process of] traditionalizing itself in flowing-static vitality. This is, as has been said, an undetermined generality, but it has in principle a structure which can be much more widely explicated by proceeding from these indications, a structure which also grounds, "implies," the possibilities for every search for and determination of concrete, factual states of affairs.

Making geometry self-evident, then, whether one is clear about this or not, is the disclosure of its historical tradition. But this knowledge, if it is not to remain empty talk or undifferentiated generality, requires the methodical production, proceeding from the present and carried out as research in the present, of differentiated self-evidences of the type discovered above (in several fragmentary investigations of what belongs to such knowledge superficially, as it were). Carried out systematically,

such self-evidences result in nothing other and nothing less than the universal a priori of history with all its highly abundant component elements.

We can also say now that history is from the start nothing other than the vital movement of the coexistence and the interweaving of original formations and sedimentations of meaning.

Anything that is shown to be a historical fact, either in the present through experience or by a historian as a fact in the past, necessarily has its *inner structure of meaning*; but especially the motivational interconnections established about it in terms of everyday understanding have deep, further and further-reaching implications which must be interrogated, disclosed. All [merely] factual history remains incomprehensible because, always merely drawing its conclusions naïvely and straightforwardly from facts, it never makes thematic the general ground of meaning upon which all such conclusions rest, has never investigated the immense structural a priori which is proper to it. Only the disclosure of the essentially general structure* lying in our present and then in every past or future historical present as such, and, in totality, only the disclosure of the concrete, historical time in which we live, in which our total humanity lives in respect to its total, essentially general structure—only this disclosure can make possible historical inquiry [*Historie*] which is truly understanding, insightful, and in the genuine sense scientific. This is the concrete, historical a priori which encompasses everything that exists as historical becoming and having-become or exists in its essential being as tradition and handing-down. What has been said was related to the total form “historical present in general,” historical time generally. But the particular configurations of culture, which find their place within its coherent historical being as tradition and as vitally handing themselves down, have within this totality only relatively self-sufficient being in traditionality, only the being of nonself-sufficient components. Correlatively, now, account would have to be taken of the subjects of historicity, the persons who create cultural formations, functioning in totality: creative personal civilization.†

* The superficial structure of the externally “ready-made” men within the social-historical, essential structure of humanity, but also the deeper [structures] which disclose the inner historicities of the persons taking part. [“Structures” is Biemel’s interpolation.]

† The historical world is, to be sure, first pregiven as a social-historical world. But it is historical only through the inner historicity of the individuals, who are individuals in their inner historicity, together with that of other communalized persons. Recall what was said in a few meager beginning expositions about memories and the constant historicity to be found in them [pp. 162f., above].

In respect to geometry one recognizes, now that we have pointed out the hiddenness of its fundamental concepts, which have become inaccessible, and have made them understandable as such in first basic outlines, that only the consciously set task of [discovering] the historical origin of geometry (within the total problem of the a priori of historicity in general) can provide the method for a geometry which is true to its origins and at the same time is to be understood in a universal-historical way; and the same is true for all sciences, for philosophy. In principle, then, a history of philosophy, a history of the particular sciences in the style of the usual factual history, can actually render nothing of their subject matter comprehensible. For a genuine history of philosophy, a genuine history of the particular sciences, is nothing other than the tracing of the historical meaning-structures given in the present, or their self-evidences, along the documented chain of historical back-references into the hidden dimension of the primal self-evidences which underlie them.* Even the very problem here can be made understandable only through recourse to the historical a priori as the universal source of all conceivable problems of understanding. The problem of genuine historical explanation comes together, in the case of the sciences, with “epistemological” grounding or clarification.

We must expect yet a second and very weighty objection. From the historicism which prevails extensively in different forms [today] I expect little receptivity for a depth-inquiry which goes beyond the usual factual history, as does the one outlined in this work, especially since, as the expression “a priori” indicates, it lays claim to a strictly unconditioned and truly apodictic self-evidence extending beyond all historical facticities. One will object: what naïveté, to seek to display, and to claim to have displayed, a historical a priori, an absolute, supertemporal validity, after we have obtained such abundant testimony for the relativity of everything historical, of all historically developed world-apperceptions, right back to those of the “primitive” tribes. Every people, large or small, has its world in which, for that people, everything fits well together, whether in mythical-magical or in European-rational terms, and in which everything can be explained perfectly. Every people has its “logic” and, accordingly, if this logic is explicated in propositions, “its” a priori.

However, let us consider the methodology of establishing historical

* But what counts as primal self-evidence for the sciences is determined by an educated person or a sphere of such persons who pose new questions, new historical questions, questions concerning the inner depth-dimension as well as those concerning an external historicity in the social-historical world.

facts in general, thus including that of the facts supporting the objection; and let us do this in regard to what such methodology presupposes. Does not the undertaking of a humanistic science of "how it really was" contain a presupposition taken for granted, a validity-ground never observed, never made thematic, of a strictly unassailable [type of] self-evidence, without which historical inquiry would be a meaningless enterprise? All questioning and demonstrating which is in the usual sense historical presupposes history [*Geschichte*] as the universal horizon of questioning, not explicitly, but still as a horizon of implicit certainty, which, in spite of all vague background-indeterminacy, is the presupposition of all determinability, or of all intention to seek and to establish determined facts.

What is historically primary in itself is our present. We always already know of our present world and that we live in it, always surrounded by an openly endless horizon of unknown actualities. This knowing, as horizon-certainty, is not something learned, not knowledge which was once actual and has merely sunk back to become part of the background; the horizon-certainty had to be already there in order to be capable of being laid out thematically; it is already presupposed in order that we can seek to know what we do not know. All not-knowing concerns the unknown world, which yet exists in advance for us *as* world, as the horizon of all questions of the present and thus also all questions which are specifically historical. These are the questions which concern men, as those who act and create in their communalized coexistence in the world and transform the constant cultural face of the world. Do we not know further—we have already had occasion to speak of this—that this historical present has its historical pasts behind it, that it has developed out of them, that historical past is a continuity of pasts which proceed from one another, each, as a past present, being a tradition producing tradition out of itself? Do we not know that the present and the whole of historical time implied in it is that of a historically coherent and unified civilization, coherent through its generative bond and constant communalization in cultivating what has already been cultivated before, whether in cooperative work or in reciprocal interaction, etc.? Does all this not announce a universal "knowing" of the horizon, an implicit knowing that can be made explicit systematically in its essential structure? Is not the resulting great problem here the horizon toward which all questions tend, and thus the horizon which is presupposed in all of them? Accordingly, we need not first enter into some kind of critical discussion of the facts set out by historicism; it is enough that even the claim of their factualness presupposes the historical a priori if this claim is to have a meaning.

But a doubt arises all the same. The horizon-exposition to which we recurred must not bog down in vague, superficial talk; it must itself arrive at its own sort of scientific discipline. The sentences in which it is expressed must be fixed and capable of being made self-evident again and again. Through what method do we obtain a universal and also fixed a priori of the historical world which is always originally genuine? Whenever we consider it, we find ourselves with the self-evident capacity to reflect—to turn to the horizon and to penetrate it in an expository way. But we also have, and know that we have, the capacity of complete freedom to transform, in thought and phantasy, our human historical existence and what is there exposed as its life-world. And precisely in this activity of free variation, and in running through the conceivable possibilities for the life-world, there arises, with apodictic self-evidence, an essentially general set of elements going through all the variants; and of this we can convince ourselves with truly apodictic certainty. Thereby we have removed every bond to the factually valid historical world and have regarded this world itself [merely] as one of the conceptual possibilities. This freedom, and the direction of our gaze upon the apodictically invariant, results in the latter again and again—with the self-evidence of being able to repeat the invariant structure at will—as what is identical, what can be made self-evident *originaliter* at any time, can be fixed in univocal language as the essence constantly implied in the flowing, vital horizon.

Through this method, going beyond the formal generalities we exhibited earlier, we can also make thematic that apodictic [aspect] of the prescientific world that the original founder of geometry had at his disposal, that which must have served as the material for his idealizations.

Geometry and the sciences most closely related to it have to do with space-time and the shapes, figures, also shapes of motion, alterations of deformation, etc., that are possible within space-time, particularly as measurable magnitudes. It is now clear that even if we know almost nothing about the historical surrounding world of the first geometers, this much is certain as an invariant, essential structure: that is was a world of "things" (including the human beings themselves as subjects of this world); that all things necessarily had to have a bodily character—although not all things could be mere bodies, since the necessarily coexisting human beings are not thinkable as mere bodies and, like even the cultural objects which belong with them structurally, are not exhausted in corporeal being. What is also clear, and can be secured at least in its essential nucleus through careful a priori explication, is that these pure bodies had spatiotemporal shapes and "mate-

rial" [*stoffliche*] qualities (color, warmth, weight, hardness, etc.) related to them. Further, it is clear that in the life of practical needs certain particularizations of shape stood out and that a technical praxis always [aimed at]³ the production of particular preferred shapes and the improvement of them according to certain directions of gradualness.

First to be singled out from the thing-shapes are surfaces—more or less "smooth," more or less perfect surfaces; edges, more or less rough or fairly "even"; in other words, more or less pure lines, angles, more or less perfect points; then, again, among the lines, for example, straight lines are especially preferred, and among the surfaces the even surfaces; for example, for practical purposes boards limited by even surfaces, straight lines, and points are preferred, whereas totally or partially curved surfaces are undesirable for many kinds of practical interests. Thus the production of even surfaces and their perfection (polishing) always plays its role in praxis. So also in cases where just distribution is intended. Here the rough estimate of magnitudes is transformed into the measurement of magnitudes by counting the equal parts. (Here, too, proceeding from the factual, an essential form becomes recognizable through a method of variation.) Measuring belongs to every culture, varying only according to stages from primitive to higher perfections. We can always presuppose some measuring technique, whether of a lower or higher type, in the essential forward development of culture, [as well as] the growth of such a technique, thus also including the art of design for buildings, of surveying fields, pathways, etc.;⁴ such a technique is always already there, already abundantly developed and pre-given to the philosopher who did not yet know geometry but who should be conceivable as its inventor. As a philosopher proceeding from the practical, finite surrounding world (of the room, the city, the landscape, etc., and temporally the world of periodical occurrences: day, month, etc.) to the theoretical world-view and world-knowledge, he has the finitely known and unknown spaces and times as finite elements within the horizon of an open infinity. But with this he does not yet have geometrical space, mathematical time, and whatever else is to become a novel spiritual product out of these finite elements which serve as material; and with his manifold finite shapes in their space-time he does not yet have geometrical shapes, the phoronomic shapes; [his shapes, as] formations developed out of praxis and thought of in terms of [gradual] perfection, clearly serve only as

³ Biemel's interpolation.

⁴ "I have reverted to the original version of this sentence as given in the critical apparatus; I can make no sense of the emended version given in the text."—D. Carr.

bases for a new sort of praxis out of which similarly named new constructions grow.

It is evident in advance that this new sort of construction will be a product arising out of an idealizing, spiritual act, one of "pure" thinking, which has its materials in the designated general pre-givens of this factual humanity and human surrounding world and creates "ideal objects" out of them.

Now the problem would be to discover, through recourse to what is essential to history [*Historie*], the historical original meaning which necessarily was able to give and did give to the whole becoming of geometry its persisting truth-meaning.

It is of particular importance now to bring into focus and establish the following insight: Only if the apodictically general content, invariant throughout all conceivable variation, of the spatiotemporal sphere of shapes is taken into account in the idealization can an ideal construction arise which can be understood for all future time and by all coming generations of men and thus be capable of being handed down and reproduced with the identical intersubjective meaning. This condition is valid far beyond geometry for all spiritual structures which are to be unconditionally and generally capable of being handed down. Were the thinking activity of a scientist to introduce something "time-bound" in his thinking, i.e., something bound to what is merely factual about his present or something valid for him as a merely factual tradition, his construction would likewise have a merely time-bound ontic meaning; this meaning would be understandable only by those men who shared the same merely factual presuppositions of understanding.

It is a general conviction that geometry, with all its truths, is valid with unconditioned generality for all men, all times, all peoples, and not merely for all historically factual ones but for all conceivable ones. The presuppositions of principle for this conviction have never been explored because they have never been seriously made a problem. But it has also become clear to us that every establishment of a historical fact which lays claim to unconditioned objectivity likewise presupposes this invariant or absolute a priori.

Only [through the disclosure of this a priori]⁵ can there be an a priori science extending beyond all historical facticities, all historical surrounding worlds, peoples, times, civilizations; only in this way can a science as *aeterna veritas* appear. Only on this fundament is based the secured capacity of inquiring back from the temporarily depleted self-evidence of a science to the primal self-evidences.

⁵ Biemel's interpolation.

Do we not stand here before the great and profound problem-horizon of reason, the same reason that functions in every man, the *animal rationale*, no matter how primitive he is?

This is not the place to penetrate into those depths themselves.

In any case, we can now recognize from all this that historicism, which wishes to clarify the historical or epistemological essence of mathematics from the standpoint of the magical circumstances or other manners of apperception of a time-bound civilization, is mistaken in principle. For romantic spirits the mythical-magical elements of the historical and prehistorical aspects of mathematics may be particularly attractive; but to cling to this merely historically factual aspect of mathematics is precisely to lose oneself to a sort of romanticism and to overlook the genuine problem, the internal-historical problem, the epistemological problem. Also, one's gaze obviously cannot then become free to recognize that facticities of every type, including those involved in the [historicist] objection, have a root in the essential structure of what is generally human, through which a teleological reason running throughout all historicity announces itself. With this is revealed a set of problems in its own right related to the totality of history and to the full meaning which ultimately gives it its unity.

If the usual factual study of history in general, and in particular the history which in most recent times has achieved true universal extension over all humanity, is to have any meaning at all, such a meaning can only be grounded upon what we can here call internal history, and as such upon the foundations of the universal historical a priori. Such a meaning necessarily leads further to the indicated highest question of a universal teleology of reason.

If, after these expositions, which have illuminated very general and many-sided problem-horizons, we lay down the following as something completely secured, namely, that the human surrounding world is the same today and always, and thus also in respect to what is relevant to primal establishment and lasting tradition, then we can show in several steps, only in an exploratory way, in connection with our own surrounding world, what should be considered in more detail for the problem of the idealizing primal establishment of the meaning-structure "geometry."

Coda